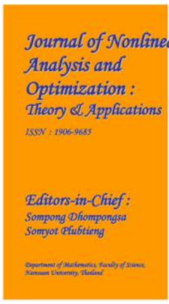
Journal of **N**onlinear **A**nalysis and **O**ptimization
Vol. 15, Issue. 1, No.1 : 2024
ISSN : **1906-9685**



OPTIMAL RESERVE INVENTORY MODELS FOR THREE CONNECTED MACHINES

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Abstract:
In the analysis of a system consisting of three machines, namely M_1, M_2, and M_3 arranged in series, the interdependence of their operations is examined. Specifically, the output of machine M_1 serves as the input to M_2, and the output of M_2 is further processed by M_3. The maintenance of machine M_1 triggers a cascade effect, causing both M_2 and M_3 to cease operation, resulting in a system-wide idle state and production loss. To mitigate this issue, reserve inventories, denoted as S_1 and S_2, are strategically placed between M_1 and M_2, and M_2 and M_3, respectively. The determination of optimal reserve inventory levels $(\widehat{S_1})$ and $(\widehat{S_2})$ takes into account associated holding costs and idle time costs. The analysis is conducted under the assumption that machine M_1 undergoes repair, with repair time characterized as a random variable satisfying the SCBZ property. This approach aims to minimize production disruptions and associated costs, offering a solution to enhance overall system efficiency.

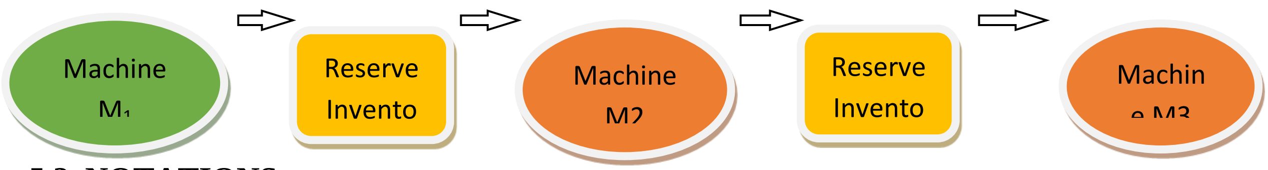
Keywords: Optimal Reserves, Repair time, Truncation point, SCBZ property, Leibnitz rule.

Introduction

Determining the optimal reserve inventory between the Machines is one of the important concepts prevailed in the inventory control theory. The method of finding the optimal reserve between the Machines find its way back to 19th century. The model initially discussed by Hanssman (1962) in which the author considered a system which contains two Machines in series and the output of Machine M_1 is the input to the Machine M_2.Whenever the Machine M_1 in the breakdown state, the Machine M_2 will be forced to be in idle state. Hence in order to avoid the disfunction of the Machine M_2. It is necessary to maintain reserve inventory between the Machines and there are two costs namely, inventory holding cost and idle time cost are involved in this system. In order to balancing out the inventory holding cost and idle time cost, an optimal reserve is needed in between the Machines. In recent research literature, there are many authors have made an attempt to study this type of problem. Rajagobal and Sathiyamoorthy (2003) have extended two Machine problem into three Machines and the authors have obtained the optimal reserve inventories between the Machines M_1 and M_2 and the Machines M_2 and M_3. Govindhan et al. (2016) have discussed the three Machines problem and in which the authors considered a system which contains three Machines such that Machine M_1 at first stage and the Machines M_2^a and M_2^b are at second stage. The output and the Machine M_1 is the simultaneous input to the Machines at second stage. Also the authors have assumed that the repair time of Machine M_1 is a random variable which undergoes change of distribution property. With these assumption the optimal reserve inventory was obtained. The change of distribution property is initially discussed by Stangl (1995). Venkatesan et al. (2016) have discussed the model for determining the optimal reserves between three Machines in series. In this model the authors have considered a system which consists three Machines in series and the output of Machine M_1 is the input to Machine M_2 and the output of the Machine M_2 is input to the Machine M_3.It is also assumed that the Machines M_2 and M_3 will be in up state during the breakdown of the Machine M_1. Hence in order to maintain the system be alive, the reserve inventories are to be maintained between the Machines M_1 and M_2 and M_2 and M_3. Under these conceptualization, the authors have obtained the optimal reserves with the assumption that the repair time of Machine M_1 is a random variable.

Selvamurugan et al. (2018) have discussed the three Machines model with the assumption that the repair time of Machine M_1 is a random variable, which follows exponential distribution and satisfies the Setting the Clock Back to Zero property. The authors also assumed that the truncation point itself a random variable and it is followed as uniform distribution, and with these assumptions the expression for optimal reserves have been obtained.

The following diagram explains the system of three machines in series.



5.2. NOTATIONS

h_1 : Cost per unit time of holding per unit of reserve inventory S_1.
h_2; Cost per unit time of holding per unit of reserve inventory S_2.
d_1 : Cost per unit time of idle time of machine M_2.
d_2 : Cost per unit time of idle time of machine M_3.
μ : Mean time interval between successive breakdowns of machine M_1 , assuming exponential distributions of inter-arrival times.
t : Continuous random variable denoting the repair time of M_1 with probability density function g(.) and CDF G (.).
r_1 : Constant consumption rate per unit time of machine M_2 from the reserve S_1.
r_2 : Constant consumption rate per unit time of machine M_3 from the reserve S_2.
S_1 : Reserve inventory between M_1 and M_2.
S_2 : Reserve inventory between M_2 and M_3.
$\widehat{S_1}$:Optimum reserve inventory between M_1 and M_2.
$\widehat{S_2}$:Optimum reserve inventory between M_2 and M_3.
T : Random variable denoting the idle time of M_2 and M_3

Main Results

This Model is a improvised one over the previous model. In this Model, it is assumed that the repair time of machine M_1 is a random variable and it undergoes change of distribution property in the sense that the repair time changes its probability distribution after a certain change point (truncation point).In doing so, it is assumed that before the truncation point the repair time distribution is exponential and it changes to Erlang(2) after the truncation point.

Hence,

$$g(t) = \begin{cases} g_1(t), & t \le x_0 \\ g_2(t), & t > x_0 \end{cases}$$

$$g_2(t) = \overline{G_1(x_0)}.\, g_2(t - x_0)$$

$$g(t) = \begin{cases} g_1(t) = \theta_1 e^{-\theta_1 t} & , if\ t \le X_0 \\ g_2(t) = e^{-\theta_1 x_0} \theta_2^2 (t - x_0) e^{-\theta_2 (t - x_0)} & , if\ \ t > X_0 \end{cases}$$

If X_0 is a random variable denoting that truncation point and it is assumed to be followed as exponential with parameter λ, then the probability density function of the repair time can be written as

$$g(t) = g_1(t)P[t \leq x_0] + g_2(t)P[\boldsymbol{t} > \boldsymbol{x_0}] \quad \textbf{(1)}$$

$$g(t) = \theta_1 e^{-\theta_1 t} e^{-\lambda t} + \int_0^t e^{-\theta_1 x_0} {\theta_2}^2 (t - x_0) e^{-\theta_2 (t-x_0)} \lambda e^{-\lambda x_0} dx_0$$

Hence, it may be observed that the expected idle time of the machine M_2 and M_3 are

$$T_{M_2} = \begin{cases} 0 & , if t \leq \dfrac{S_1}{r_1} \\ t - \dfrac{S_1}{r_1} & , if t > \dfrac{S_1}{r_1} \end{cases}$$

$$T_{M_3} = \begin{cases} t - \dfrac{S_1}{r_1} - \dfrac{S_2}{r_2}, & t > \left(\dfrac{S_1}{r_1} + \dfrac{S_2}{r_2}\right) \\ 0, & t \leq \left(\dfrac{S_1}{r_1} + \dfrac{S_2}{r_2}\right) \end{cases}$$

Thus, the total expected cost is given as

$$E(C) = h_1 S_1 + h_2 S_2 + \frac{d_1}{\mu}\left\{E\left(T_{M_2}\right)\right\} + \frac{d_2}{\mu}\left\{E\left(T_{M_3}\right)\right\} \quad \textbf{(2)}$$

$$E(T_{M_2}) = \int_{\frac{S_1}{r_1}}^{\infty} \left(t - \frac{S_1}{r_1}\right) g(t) dt$$

$$= \int_{\frac{S_1}{r_1}}^{\infty} \left(t - \frac{S_1}{r_1}\right) \left\{\theta_1 e^{-\theta_1 t} e^{-\lambda t} + \int_0^t e^{-\theta_1 x_0} {\theta_2}^2 (t - t_0) e^{-\theta_2 (t-x_0)} \lambda e^{-\lambda x_0} dx_0\right\} dt$$

$$E\left(T_{M_2}\right) = I_1 + \frac{\lambda {\theta_2}^2}{(\lambda + \theta_1 - \theta_2)} \int_{\frac{S_1}{r_1}}^{\infty} t\left(t - \frac{S_1}{r_1}\right) e^{-\theta_2 t} dt + \frac{\lambda {\theta_2}^2}{(\lambda + \theta_1 - \theta_2)^2} \int_{\frac{S_1}{r_1}}^{\infty} \left(t - \frac{S_1}{r_1}\right) e^{-t(\lambda + \theta_1)} dt$$

$$- \frac{\lambda {\theta_2}^2}{(\lambda + \theta_1 - \theta_2)^2} \int_{\frac{S_1}{r_1}}^{\infty} \left(t - \frac{S_1}{r_1}\right) e^{-t\theta_2} dt$$

$$E\left(T_{M_2}\right) = I_1 + I_2 + I_3 - I_4 \quad \textbf{(3)}$$

$$E\left(T_{M_3}\right) = \int_{\frac{S_1}{r_1} + \frac{S_2}{r_2}}^{\infty} \left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right) g(t) dt$$

$$E\left(T_{M_3}\right) = \int_{\frac{S_1}{r_1} + \frac{S_2}{r_2}}^{\infty} \left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right) \left\{\theta_1 e^{-t(\lambda + \theta_1)} + {\theta_2}^2 \lambda e^{-t\theta_2} \int_0^t (t - t_0) e^{-x_0(\lambda + \theta_1 - \theta_2)} dx_0\right\} dt$$

$$E\left(T_{M_3}\right) = I_5 + \frac{\theta_2^2 \lambda}{(\lambda + \theta_1 - \theta_2)^2} \int_{\frac{S_1}{r_1} + \frac{S_2}{r_2}}^{\infty} \left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right) e^{-t(\lambda + \theta_1)} dt$$

$$- \frac{\theta_2^2 \lambda}{(\lambda + \theta_1 - \theta_2)^2} \int_{\frac{S_1}{r_1} + \frac{S_2}{r_2}}^{\infty} \left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right) e^{-t\theta_2} dt$$

$$E\left(T_{M_3}\right) = I_5 + I_6 - I_7 \quad \textbf{(4)}$$

It can be rewritten as

$$E(C) = \mathrm{h}_1 S_1 + \mathrm{h}_2 S_2 + \frac{d_1}{\mu}\left\{E\left(T_{M_2}\right)\right\} + \frac{d_2}{\mu}\left\{E\left(T_{M_3}\right)\right\}$$

$$\frac{dE(C)}{dS_1} = 0$$

$$\Rightarrow \mathrm{h}_1 + \frac{d_1}{\mu}\left\{\frac{dE\left(T_{M_2}\right)}{dS_1}\right\} + \frac{d_2}{\mu}\left\{\frac{dE\left(T_{M_3}\right)}{dS_1}\right\} = 0$$

$$\Rightarrow \mathrm{h}_1 + \frac{d_1}{\mu}\left\{\frac{dI_1}{dS_1} + \frac{dI_2}{dS_1} + \frac{dI_3}{dS_1} - \frac{dI_4}{dS_1}\right\} + \frac{d_2}{\mu}\left\{\frac{dI_5}{dS_1} + \frac{dI_6}{dS_1} - \frac{dI_7}{dS_1}\right\} = 0 \quad \textbf{(5)}$$

From (3)

$$\frac{dE\left(T_{M_2}\right)}{dS_1} = \frac{dI_1}{dS_1} + \frac{dI_2}{dS_1} + \frac{dI_3}{dS_1} - \frac{dI_4}{dS_1}$$

$$\frac{dI_1}{dS_1} = \frac{d}{dS_1}\int_{\frac{S_1}{r_1}}^{\infty} \theta_1\left(t - \frac{S_1}{r_1}\right)e^{-t(\lambda+\theta_1)}dt$$

$$\frac{dI_1}{dS_1} = -\frac{\theta_1 e^{-\frac{S_1}{r_1}(\lambda+\theta_1)}}{r_1(\lambda+\theta_1)} \quad \textbf{(6)}$$

$$\frac{dI_2}{dS_1} = \frac{d}{dS_1}\left(\frac{\lambda\theta_2^2}{\lambda+\theta_1-\theta_2}\right)\int_{\frac{S_1}{r_1}}^{\infty} t\left(t - \frac{S_1}{r_1}\right)e^{-\theta_2 t}dt$$

$$\frac{dI_2}{dS_1} = -\frac{\lambda{\theta_2}^2}{r_1(\lambda+\theta_1-\theta_2)}\left\{\left(\frac{S_1}{r_1\theta_2}e^{-\theta_2\frac{S_1}{r_1}} + \frac{1}{{\theta_2}^2}e^{-\theta_2\frac{S_1}{r_1}}\right)\right\} \quad \textbf{(7)}$$

$$\frac{dI_3}{dS_1} = \frac{d}{ds_1}\frac{\lambda{\theta_2}^2}{(\lambda+\theta_1-\theta_2)^2}\int_{\frac{S_1}{r_1}}^{\infty}\left(t - \frac{S_1}{r_1}\right)e^{-t(\lambda+\theta_1)}dt$$

$$\frac{dI_3}{dS_1} = -\frac{\lambda{\theta_2}^2}{r_1(\lambda+\theta_1-\theta_2)^2}\left[\frac{e^{-\frac{S_1}{r_1}(\lambda+\theta_1)}}{(\lambda+\theta_1)}\right] \quad \textbf{(8)}$$

$$\frac{dI_4}{dS_1} = \frac{\lambda{\theta_2}^2}{(\lambda+\theta_1-\theta_2)^2}\int_{\frac{S_1}{r_1}}^{\infty}\left(t - \frac{S_1}{r_1}\right)e^{-\theta_2 t}dt$$

$$\frac{dI_4}{dS_1} = -\frac{\lambda{\theta_2}^2}{(\lambda+\theta_1-\theta_2)^2}\frac{e^{-\frac{S_1}{r_1}\theta_2}}{r_1\theta_2} \quad \textbf{(9)}$$

Hence,

$$\frac{dE(T_{M_2})}{dS_1} = -\theta_1\frac{e^{-\frac{S_1}{r_1}(\lambda+\theta_1)}}{r_1(\lambda+\theta_1)} - \frac{\lambda{\theta_2}^2\frac{S_1}{r_1}e^{-\frac{S_1}{r_1}\theta_2}}{r_1\theta_2(\lambda+\theta_1-\theta_2)} - \frac{\lambda{\theta_2}^2 e^{-\frac{S_1}{r_1}\theta_2}}{r_1{\theta_2}^2(\lambda+\theta_1-\theta_2)} - \frac{\lambda{\theta_2}^2 e^{-\frac{S_1}{r_1}(\lambda+\theta_1)}}{r_1(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)} + \frac{\lambda{\theta_2}^2}{(\lambda+\theta_1-\theta_2)^2 r_1\theta_2}e^{-\frac{S_1}{r_1}\theta_2} \quad (\mathbf{10})$$

From (4)

$$\frac{dI_5}{dS_1} = \frac{d}{ds_1}\left(\theta_1\int_{\frac{S_1}{r_1}+\frac{S_2}{r_2}}^{\infty}\left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right)e^{-t(\lambda+\theta_1)}dt\right)$$

$$\frac{dI_5}{dS_1} = -\frac{\theta_1 e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_1(\lambda+\theta_1)}$$

$$\frac{dI_6}{dS_1} = \frac{d}{ds_1}\left(\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\int_{\frac{S_1}{r_1}+\frac{S_2}{r_2}}^{\infty}\left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right)e^{-t(\lambda+\theta_1)}dt\right)$$

$$\frac{dI_6}{dS_1} = \frac{-{\theta_2}^2\lambda e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_1(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}$$

$$\frac{dI_7}{dS_1} = \frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\int_{\frac{S_1}{r_1}+\frac{S_2}{r_2}}^{\infty}\left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right)e^{-\theta_2 t}dt$$

$$\frac{dI_7}{dS_1} = \frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\frac{e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)\theta_2}}{r_1\theta_2}$$

Hence,

$$\frac{dE\left(T_{M_3}\right)}{dS_1}=-\frac{\theta_1 e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_1(\lambda+\theta_1)}-\frac{{\theta_2}^2\lambda e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_1(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}+\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\frac{e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)\theta_2}}{r_1\theta_2} \quad \mathbf{(11)}$$

$$\frac{dE(C)}{dS_1}=0$$

$$\Rightarrow \mathrm{h}_1+\frac{d_1}{\mu}\left\{\frac{dE\left(T_{M_2}\right)}{dS_1}\right\}+\frac{d_2}{\mu}\left\{\frac{dE\left(T_{M_3}\right)}{dS_1}\right\}=0 \quad \mathbf{(12)}$$

Substituting (10) and (11) in (12) the resultant equation is

$$=h_1+\frac{d_1}{\mu}\left\{-\theta_1\frac{e^{-\frac{S_1}{r_1}(\lambda+\theta_1)}}{r_1(\lambda+\theta_1)}-\frac{\lambda{\theta_2}^2\frac{S_1}{r_1}e^{-\frac{S_1}{r_1}\theta_2}}{r_1\theta_2(\lambda+\theta_1-\theta_2)}-\frac{\lambda{\theta_2}^2e^{-\frac{S_1}{r_1}\theta_2}}{r_1{\theta_2}^2(\lambda+\theta_1-\theta_2)}-\frac{\lambda{\theta_2}^2e^{-\frac{S_1}{r_1}(\lambda+\theta_1)}}{r_1(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}+\frac{\lambda{\theta_2}^2}{(\lambda+\theta_1-\theta_2)^2r_1\theta_2}e^{-\frac{S_1}{r_1}\theta_2}\right\}+\frac{d_2}{\mu}\left\{-\frac{\theta_1 e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_1(\lambda+\theta_1)}-\frac{{\theta_2}^2\lambda e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_1(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}+\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\frac{e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)\theta_2}}{r_1\theta_2}\right\}=0 \quad \mathbf{(13)}$$

And in similar way, $\boldsymbol{\frac{dE(C)}{dS_2}=0}$

$$\Rightarrow \mathrm{h}_2+\frac{d_1}{\mu}\left\{\frac{dE\left(T_{M_2}\right)}{dS_2}\right\}+\frac{d_2}{\mu}\left\{\frac{dE\left(T_{M_3}\right)}{dS_2}\right\}=0 \quad \mathbf{(14)}$$

$$\Rightarrow \mathrm{h}_2+\frac{d_1}{\mu}\left\{\frac{dI_1}{dS_2}+\frac{dI_2}{dS_2}+\frac{dI_3}{dS_2}-\frac{dI_4}{dS_2}\right\}+\frac{d_2}{\mu}\left\{\frac{dI_5}{dS_2}+\frac{dI_6}{dS_2}-\frac{dI_7}{dS_2}\right\}=0 \quad \mathbf{(15)}$$

Since S_2 is not involved in $\frac{dE\left(T_{M_2}\right)}{dS_2}$

Hence, $\frac{dE\left(T_{M_2}\right)}{dS_2}=0$ **(16)**

$$\frac{dE\left(T_{M_3}\right)}{dS_2}=\frac{dI_5}{dS_2}+\frac{dI_6}{dS_2}-\frac{dI_7}{dS_2} \quad \mathbf{(17)}$$

$$\frac{dI_5}{dS_2}=\frac{d}{ds_2}\left(\theta_1\int_{\frac{S_1}{r_1}+\frac{S_2}{r_2}}^{\infty}\left(t-\frac{s_1}{r_1}-\frac{s_2}{r_2}\right)e^{-t(\lambda+\theta_1)}dt\right)$$

$$\frac{dI_5}{dS_2}=-\frac{\theta_1 e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_2(\lambda+\theta_1)}$$

$$\frac{dI_6}{dS_2}=\frac{d}{ds_2}\left(\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\int_{\frac{S_1}{r_1}+\frac{S_2}{r_2}}^{\infty}\left(t-\frac{s_1}{r_1}-\frac{s_2}{r_2}\right)e^{-t(\lambda+\theta_1)}dt\right)$$

$$=\frac{dI_6}{dS_2}=\frac{-{\theta_2}^2\lambda e^{-\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)(\lambda+\theta_1)}}{r_2(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}$$

$$\frac{dI_7}{dS_2} = \frac{d}{ds_2}\left(\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\int_{\frac{s_1}{r_1}+\frac{s_2}{r_2}}^{\infty}\left(t-\frac{s_1}{r_1}-\frac{s_2}{r_2}\right)e^{-\theta_2 t}dt\right)$$

$$\frac{dI_7}{dS_2} = -\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\frac{e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta_2}}{r_2\theta_2}$$

$$\frac{dE(T_{M_3})}{dS_2} = \frac{dI_5}{dS_1}+\frac{dI_6}{dS_1}-\frac{dI_7}{dS_1}$$

Hence,

$$\frac{dE(T_{M_3})}{dS_2} = -\frac{\theta_1 e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)(\lambda+\theta_1)}}{r_2(\lambda+\theta_1)}-\frac{{\theta_2}^2\lambda e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)(\lambda+\theta_1)}}{r_2(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}+\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\frac{e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta_2}}{r_2\theta_2} \quad \textbf{(18)}$$

Substituting (16) and (18) in (14) the resultant equation is

$$\Rightarrow \mathrm{h}_2+\frac{d_1}{\mu}\{0\}+\frac{d_2}{\mu}\left\{-\frac{\theta_1 e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)(\lambda+\theta_1)}}{r_2(\lambda+\theta_1)}-\frac{{\theta_2}^2\lambda e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)(\lambda+\theta_1)}}{r_2(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}+\frac{{\theta_2}^2\lambda}{(\lambda+\theta_1-\theta_2)^2}\frac{e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta_2}}{r_2\theta_2}\right\}=0 \quad (\mathbf{19})$$

Solving the (13) and (19) the resultant equation is

$$h_1r_1-h_2r_2+\frac{d_1}{\mu}\left\{-\theta_1\frac{e^{-\frac{s_1}{r_1}(\lambda+\theta_1)}}{(\lambda+\theta_1)}-\frac{\lambda{\theta_2}^2\frac{s_1}{r_1}e^{-\frac{s_1}{r_1}\theta_2}}{\theta_2(\lambda+\theta_1-\theta_2)}-\frac{\lambda{\theta_2}^2e^{-\frac{s_1}{r_1}\theta_2}}{{\theta_2}^2(\lambda+\theta_1-\theta_2)}-\frac{\lambda{\theta_2}^2e^{-\frac{s_1}{r_1}(\lambda+\theta_1)}}{(\lambda+\theta_1-\theta_2)^2(\lambda+\theta_1)}+\frac{\lambda{\theta_2}^2}{(\lambda+\theta_1-\theta_2)^2\theta_2}e^{-\frac{s_1}{r_1}\theta_2}\right\} \quad \textbf{(20)}$$

Using the equations (19) and (20) numerically by taking fixed values for h_1, h_2, r_1, r_2, d_1, d_2, μ, λ, θ_1 *and* θ_2 the optimal value of $\widehat{S_1}$ and $\widehat{S_2}$ can be obtained.

Numerical Illustrations

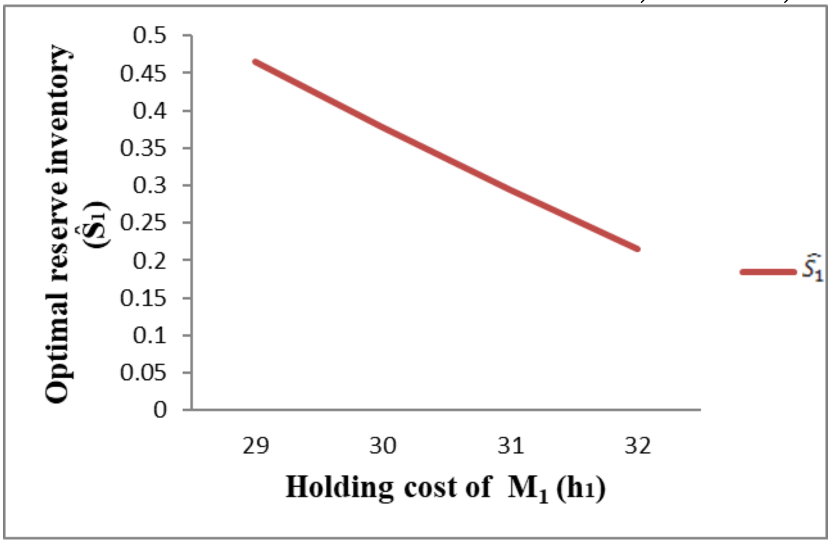
The variations, in the values of $\widehat{S_1}$ and $\widehat{S_2}$, consequent to the changes in the parameter h_1, h_2, r_1, r_2, d_1, d_2, μ, λ, θ_1 and θ_2 have been studied by taking the numerical illustrations. The tables and the corresponding graphs are given below.

Case (i)

Since the holding cost h_1 is related to S_1, for the fixed values of $h_2 = 10$, $d_1 = 50$, $d_2 = 60$, $r_1 = 2$, $r_2 = 2$, θ_1,=1.2, θ_2,=1.2 $\mu = 1.0$ and $\lambda = 1$. The variations, in the values of the optimal reserve inventory $\widehat{S_1}$ for various values of h_1, is shown in the table.

The Behaviour of optimal reserve inventory for the changes in the value of holding cost of M_1

h_1	29	30	31	32
$\widehat{S_1}$	0.4659	0.3776	0.2941	0.2149



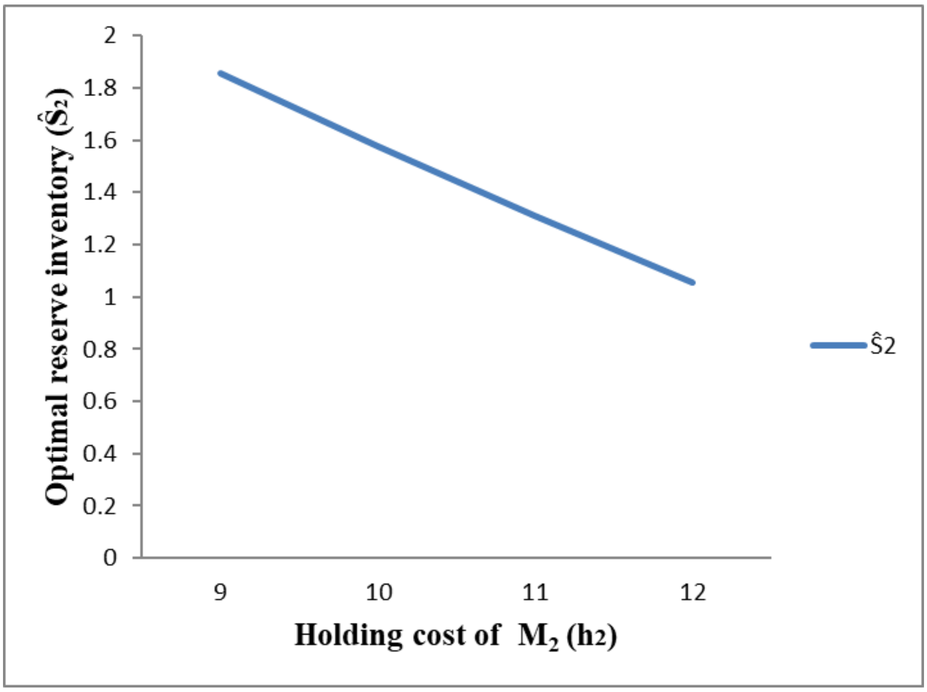
The Behaviour of optimal reserve inventory for the changes in the values of holding cost of M1

Case (ii)

The fixed values of $h_1 = 30$, $d_1 = 50$, $d_2 = 60$, $r_1 = 2$, $r_2 = 2$, θ_1,=1.2, θ_2,=1.2, $\mu = 1.0$ and $\lambda = 1$, and the variations, in the values of the optimal reserve inventory $\widehat{S_2}$ for various values of h_2 are shown in the table.

The Behaviour of optimal reserve inventory for the changes in the value of holding cost of M2

h_2	9	10	11	12
$\widehat{S_2}$	1.8564	1.5752	1.3094	1.0547



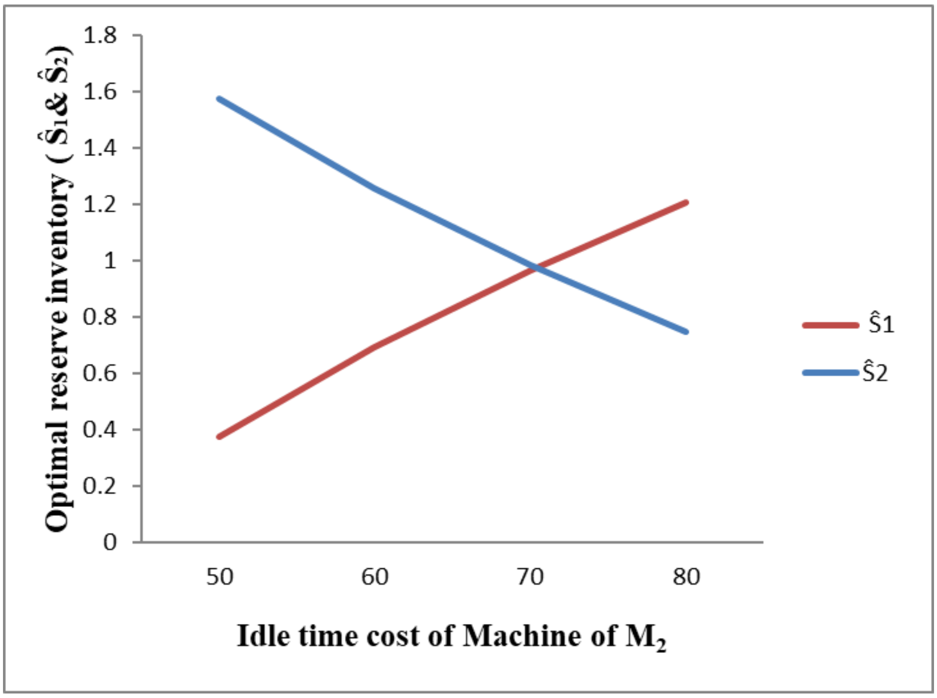
The Behaviour of optimal reserve inventory for the changes in the value of holding cost of M2

Case (iii)

The fixed values of $h_1 = 30$, $h_2 = 10$, $d_2 = 60$, $r_1 = 2$, $r_2 = 2$, θ_1,=1.2, θ_2,=1.2, $\mu = 1.0$ and $\lambda = 1$, and the variations, in the values of the optimal reserve inventory $\widehat{S_1}$ and $\widehat{S_2}$ for various values of d_1, are shown in the table.

The Behaviour of optimal reserve inventory for the changes in the values of idle time cost of Machine M_2

d_1	50	60	70	80
$\widehat{S_1}$	0.3776	0.694	0.9669	1.2067
$\widehat{S_2}$	1.5752	1.2588	0.9859	0.7461



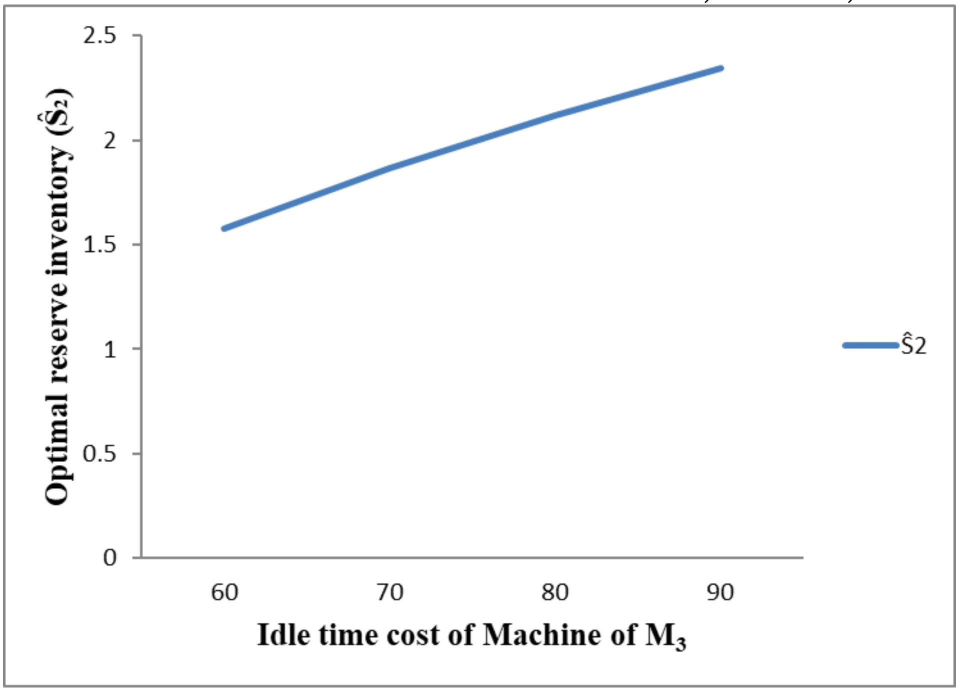
The Behaviour of optimal reserve inventory for the changes in the values of idle time cost of Machine M_2

Case (iv)

The fixed values of $h_1 = 30$, $h_2 = 10$, $d_1 = 50$, $r_1 = 2$, $r_2 = 2$, θ_1,=1.2, θ_2,=1.2, $\mu = 1.0$ and $\lambda = 1$, and the variations in the values of the optimal reserve inventory $\widehat{S_2}$ for various values of d_2, are shown in the table.

The Behaviour of optimal reserve inventory for the changes in the value of idle time of M_3

d_2	60	70	80	90
$\widehat{S_2}$	1.5752	1.8648	2.1179	2.3426



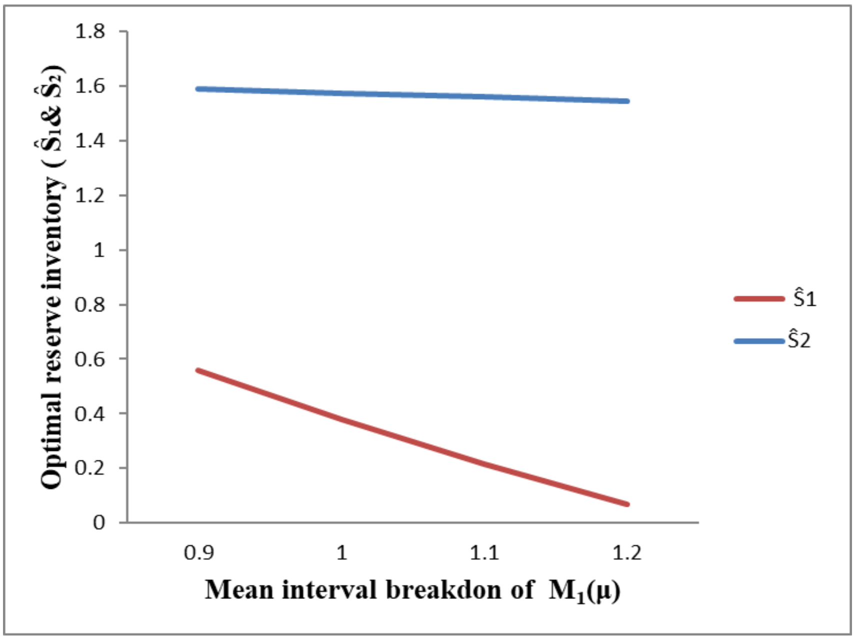
The Behaviour of optimal reserve inventory for the changes in the value of idle time of M3

Case (v)

The fixed values of $h_1 = 30$, $h_2 = 10$, $d_1 = 50$, $d_2 = 60$, $r_1 = 2$, $r_2 = 2$, θ_1,=1.2, θ_2,=1.2 and $\lambda = 1$, and the variations, in the values of the optimal reserve inventory $\widehat{S_1}$ and $\widehat{S_2}$ for various values of μ are shown in the table.

The Behaviour of optimal reserve inventory for the changes in the value of mean interval breakdown of M1

μ	0.9	1.0	1.1	1.2
$\widehat{S_1}$	0.5596	0.3776	0.2149	0.0682
$\widehat{S_2}$	1.5909	1.5752	1.5604	1.5461



TheBehaviour of optimal reserve inventory for the changes in the value of mean interval breakdown of M1

Conclusions

From the tables and graphs, it is observed that

i) As the inventory holding cost ‘h_1’ increases, the value of $\widehat{S_1}$ decreases suggesting a smaller reserve between the Machines M_1 and M_2.

ii) If the inventory holding cost ‘h_2’ increases, the value of $\widehat{S_2}$ will decreases to suggest smaller reserve between the Machines M_2 and M_3.

iii) If the d_1 of the idle time cost of Machine M_2 increases, a higher level o $\widehat{S_1}$f is suggested. However, the $\widehat{S_2}$ is found to be decreased due to the reason that the optimal reserve $\widehat{S_2}$ is more, as it supports the Machines M_2 and M_3.

iv) As the d_2 idle time cost of inventory increases a larger inventory of $\widehat{S_1}$ is suggested.

v) As the parameter μ, the mean inter-arrival time between successive breakdowns of Machines M_1 is in increase then the optimal reserve inventories are in decrease. It suggested to have smaller inventories of $\widehat{S_1}$ and $\widehat{S_2}$

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